

**10-th CHICO TOPOLOGY  
CONFERENCE SCHEDULE  
June 1, 2, 3, 2006**

Thursday, June 1<sup>st</sup> – Holt 185

- 9:20 a.m.      Welcome  
Paul Zingg, President,  
California State University, Chico
- 9:30 - 10:20 a.m.      “Continuous Images of Plane Continua”  
James T. Rogers, Jr., Tulane University
- 10:30 - 10:50 a.m.      “Characterizations of the Pseudo-Arc”  
Wayne Lewis, Texas Tech University
- 11:00 - 11:20 a.m.      “More on continua for which the set function  
T is continuous”  
Sergio Macias, Universidad Nacional  
Autonoma de Mexico
- 11:30 – 11:50 a.m.      “Centers and Shore Points of a Dendroid”  
Van C. Nall, University of Richmond
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- 1:30 - 2:20 p.m.      “A second update on the elusive  
fixed-point property”  
Charles L. Hagopian, California State  
University, Sacramento
- 2:30 - 2:50 p.m.      “An uncountable family of semismooth  
non-smooth dendroids”  
Jorge M. Martinez Montejano, Universidad  
Nacional Autonoma de Mexico
- 3:00 - 3:20 p.m.      “Aposyndesis in plane continua without disjoint  
subcontinua with nonvoid interiors”  
Eldon Vought, California State University,  
Chico

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Friday, June 2<sup>nd</sup> – Holt 185

- 9:30 - 10:20 a.m. “Means on continua”  
Alejandro Illanes, Universidad Nacional  
Autonoma de Mexico
- 10:30 - 10:50 a.m. “Dendrites without unique hyperspace”  
Gerardo Acosta, Universidad Nacional  
Autonoma de Mexico
- 11:00 - 11:20 a.m. “Functions that separate the cross-space”  
Kenneth R. Kellum, San Jose State  
University
- 11:30 - 11:50 a.m. “Even-to-1 mappings on manifolds and fixed  
points”  
M.M. Marsh, California State University,  
Sacramento

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- 1:30 - 2:20 p.m. “Expansive homeomorphisms and entropy”  
Christopher Mouron, Rhodes College
- 2:30 - 2:50 p.m. “The Sarkovskii Order for periodic continua II”  
David Ryden, Baylor University
- 3:00 - 3:20 p.m. “The  $\Omega$  EP – property on continua”  
Veronica Martínez de la Vega y Mansilla,  
Universidad Nacional Autonoma de Mexico

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Saturday, June 3<sup>rd</sup> – Holt 185

- 9:30 - 10:20 a.m. “Recent study in homogeneous continua”  
Janusz Prajs, California State University,  
Sacramento
- 10:30 - 10:50 a.m. “Explosion points and functions with  
indecomposable inverse limits”  
Michael Siler, Rhodes College
- 11:00 - 11:20 a.m. “Another triangle inequality”  
Hidefumi Katsuura, San Jose State  
University

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Title. Dendrites without unique hyperspace.

Gerardo Acosta

Abstract. In this talk we show examples of dendrites  $X$  for which there exists a dendrite  $Y$  such that  $X$  and  $Y$  are not homeomorphic, while the hyperspaces  $C(X)$  and  $C(Y)$  of subcontinua of  $X$  and  $Y$  are homeomorphic. By a result of D. Herrera, the set of end-points of those dendrites  $X$  and  $Y$  is not closed.

## A SECOND UPDATE ON THE ELUSIVE FIXED-POINT PROPERTY

Charles L. Hagopian

California State University, Sacramento

Abstract. R.H. Bing's 1969 Monthly article, "The elusive fixed point property," has been an invaluable guide to generations of mathematicians. Bing was interested in the fundamental problem of determining which continua have the fixed-point property. His article consists of twelve questions and a variety of related theorems and examples. At the 2006 Spring Topology and Dynamical Systems Conference in Greensboro, I presented a survey paper based on Bing's questions, some results that followed the publication of his article, and some related unsolved problems. Recently, Roman Manka sent me a preprint of his survey paper on Bing's twelve questions. It contains several interesting facts and problems that were not included in my paper. I will discuss some of this information in a revised summary. Although considerable progress has been made in this area of fixed-point theory, several general problems remain unsolved. It is still unknown whether every plane continuum that does not separate the plane has the fixed-point property.

## MEANS ON CONTINUA

Alejandro Illanes, Instituto de Matemáticas, UNAM.

Given a continuum  $X$ , a *mean* is a continuous function  $m : X \times X \rightarrow X$  such that  $m((p, q)) = m((q, p))$  for each  $p, q \in X$  and  $m(p, p) = p$  for each  $p \in X$ .

A natural problem related to this concept is which continua admit a mean. No characterization is known yet. In this talk we discuss some partial results on this problem. In particular, we discuss the recent advances on the problem: Is the arc the only arc-like continuum which admits a mean?

Another Triangle Inequality

Hidefumi Katsuura

San Jose State University

**Abstract:** Let  $a_i, b_i > 0$ ,  $c_i = a_i + b_i$ ,  $i = 1, 2, 3$ . The triangle inequality states that

$$(c_1^2 + c_2^2 + c_3^2)^{1/2} \geq (a_1^2 + a_2^2 + a_3^2)^{1/2} + (b_1^2 + b_2^2 + b_3^2)^{1/2}.$$

There is an interesting other side to this inequality. We will prove that

$$(c_1^{-2} + c_2^{-2} + c_3^{-2})^{-1/2} \geq (a_1^{-2} + a_2^{-2} + a_3^{-2})^{-1/2} + (b_1^{-2} + b_2^{-2} + b_3^{-2})^{-1/2}.$$

### Functions that separate the cross-space

Kenneth R. Kellum

San José State University

In a recent paper, M. R. Wojcik and M. S. Wojcik give an interesting new characterization of continuity for real-valued functions, purely in terms of connectedness. They prove that if  $X$  is a connected space then  $f : X \rightarrow \mathfrak{R}$  is continuous if and only if the graph of  $f$  is connected and the complement of the graph of  $f$  is disconnected.

We explore further the consequences of the complement of a function's graph being disconnected and extend these results. Suppose  $X$  is connected and  $f : X \rightarrow \mathfrak{R}$  has the property that its complement is disconnected. We prove that if  $f$  is bounded, then  $f$  is continuous. Also, we prove that if  $f$  has the Gibson property, that is,  $f(\text{cl}(U)) \subset \text{cl}(f(U))$  for each open set  $U$ , then  $f$  is continuous.

## CHARACTERICATIONS OF THE PSEUDO-ARC

Wayne Lewis

Texas Tech University

The pseudo-arc has several different characterizations. Some of these are well known. Some are less well known. There are perhaps some that are not yet known, though some possible ones have been conjectured. We shall discuss some of these characterizations or possible characterizations and their significance.

Title: More on continua for which the set function  $T$  is continuous.

Sergio Macias

Abstract:

Let  $X$  be a continuum, and let  $P(X)$  denote the power set of  $X$ . Define  $T: P(X) \rightarrow P(X)$  by  $T(A) = X \setminus \{x \in X \mid \text{there exists a subcontinuum } W \text{ of } X \text{ such that } x \text{ is in } \text{Int}(W) \text{ and } W \text{ and } A \text{ have no points in common}\}$ . We say that  $X$  is point  $T$ -symmetric provided that given two points  $p$  and  $q$  of  $X$ , we have that  $p$  is in  $T(\{q\})$  if and only if  $q$  is in  $T(\{p\})$ . We say that  $T$  is continuous for  $X$  provided that the restriction of  $T$  to the hyperspace  $2^X$  (with the Hausdorff metric) is continuous. The purpose of the talk is to show the following: Theorem. Let  $X$  be a point  $T$ -symmetric continuum for which  $T$  is continuous. Then  $G = \{T(\{x\}) \mid x \text{ is in } X\}$  is a continuous decomposition of  $X$  such that  $X/G$  is a locally connected continuum. This gives a partial answer to a question of David Bellamy (Houston Problem Book Problem 162).

## Even-to-1 Mappings on Manifolds and Fixed Points

M. M. Marsh

California State University, Sacramento

H. Katsuura has shown that each even-to-one mapping on the unit circle has a fixed point and he has asked if each such mapping on an annulus must have a fixed point. We establish a relationship between the "degree" of a mapping and  $k$  for  $k$ -to-1 maps of orientable manifolds. Using the degree theory, we generalize Katsuura's result to some other manifolds that do not have the fixed point property. Unfortunately, we are not able to answer his question for annuli since the degree notion is not well-behaved for manifolds with boundary.

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Title: The  $\Omega EP$ -Property on Continua.

Veronica Martínez de la Vega y Mansilla.

Instituto de Matemáticas, UNAM,

México

Abstract: Let  $X$  be a continuum and  $f: X \rightarrow X$  a map. We define  $EP(f) = \{x \in X : \exists n, m \in \mathbb{N} \text{ such that } f^n(x) = f^{m+n}(x)\}$  and  $\Omega(f) = \{x \in X : \text{for every open set } U \text{ of } X, x \in U, \text{there exists } y \in U \text{ and } n \in \mathbb{N} \text{ such that } f^n(y) \in U\}$ , we say that  $X$  has the  $\Omega EP$ -Property provided that  $\Omega EP \subset \text{cl}_X EP(f)$ . In this talk we show that a locally connected continua has the  $\Omega EP$ -property iff it is a dendrite and does not contain a homeomorphic copy of the null-comb. Also we prove that a nonlocally connected continua containing an arc  $A$  and a point  $p \in A$  such that  $X$  is not connected in kleinen at  $p$ , does not have the  $\Omega EP$ -property. Though there are nonlocally connected continua with

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Title: An uncountable family of smooth non smooth dendroids.

Jorge M. Martínez Montejano

Instituto de Matemáticas, UNAM,

México

Abstract. A dendroid  $X$  is smooth provided that there exists a point  $p \in X$  such that for every convergent sequence  $\{x_n\}_{n=1}^{\infty} \subset X$ ,  $px_n \rightarrow px$ . A dendroid  $X$  is semi-smooth provided that there exists a point  $p \in X$  such that for every convergent sequence  $\{x_n\}_{n=1}^{\infty} \subset X$ ,  $\limsup px_n$  is an arc. In 1970, J. J. Charatonik and C. Eberhart asked if the property of non-smoothness is finite in the class of semi-smooth dendroids. In this talk we answer this question negatively by showing an uncountable family  $\{X_\alpha\}_{\alpha \in I}$  of semi-smooth non-smooth dendroids with the following property; if  $A$  is a non-locally connected subcontinuum of  $X_\alpha$ , then  $A$  cannot be embedded in  $X_\beta$  for  $\alpha \neq \beta$ . The family  $\{X_\alpha\}_{\alpha \in I}$  can be chosen such that each  $X_\alpha$  has the following properties;  $X_\alpha$  is a planar fan with a closed set of end points.

CHRISTOPHER MOURON

ABSTRACT. A homeomorphism  $h : X \rightarrow X$  is called *expansive* provided that for some fixed  $c > 0$  and every  $x, y \in X$  there exists an integer  $n$ , dependent only on  $x$  and  $y$ , such that  $d(h^n(x), h^n(y)) > c$ . Entropy is a measure of how fast points move apart under an iterated homeomorphism. I discuss how the topology must be rich in order for a continuum to admit an expansive or a positive entropy homeomorphism and the relationship between the two. Then these concepts will be generalized to group actions where some new and surprising results will be presented.

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## Centers and Shore Points of a Dendroid

Van C. Nall

### Abstract

A point  $p$  in a continuum  $X$  is a *center* for  $X$  if there are two points  $b$  and  $c$  in  $X$ , such that for every  $\epsilon > 0$  there is a continuum  $C$  containing  $p$  of diameter less than  $\epsilon$ , and there are open sets  $U$  and  $V$ , containing  $b$  and  $c$  respectively such that every continuum in  $X$  from  $U$  to  $V$  intersects  $C$ . A subset  $A$  of a continuum  $X$  is a *shore set* if there is a sequence of continua from  $X \setminus A$  converging to  $X$ . We explore the relationship between centers and shore sets in dendroids and  $\lambda$ -dendroids.

# Recent Study in Homogeneous Continua

Janusz R. Prajs

In this talk I will present my recent study in homogeneous continua. In joint work with Keith Whittington, a number of new concepts were found particularly useful in the classifying study of homogeneous continua. Among them there are the concepts of *filament* and *ample* subcontinua, *filament composant*, and *filament additive continuum*. This talk will be devoted to my recent results that involve these new ideas. Part of this work was obtained in cooperation with Keith Whittington.

## Continuous Images of Plane Continua

James T. Rogers, Jr.

In the last several years continuum theory has been used to great advantage in holomorphic dynamics. Some of the theorems that have been used are "pure continuum theory." We discuss a few of these and see if we can apply them to a continuum theory problem.

## The Sharkovskii Order for Periodic Continua II

David Ryden

Suppose  $f$  is a map of a hereditarily decomposable continuum  $X$  onto itself. A *periodic continuum* of  $f$  is a subcontinuum  $K$  of  $X$  such that  $f^n[K] = K$  for some  $n$ , and  $K$  is *maximal* provided it is a proper subcontinuum of  $X$  and the only periodic continuum that properly contains it is  $X$ . Building on a result of Minc and Transue, the speaker has shown that, apart from a trivial case, the maximal periodic continua of  $f$  follow the Sharkovskii order. Sharkovskii also showed that, for every terminal segment in the Sharkovskii order, there is an interval map for which the set of all periodic points includes a point of period  $n$  if and only if  $n$  belongs to the prescribed terminal segment. That this is not the case for the maximal periodic continua of  $f$  will be the focus of this talk.

# Explosion Points and Functions with Indecomposable Inverse Limits

MICHAEL SILER

ABSTRACT. Let  $X$  be a continuum and  $f : X \rightarrow X$  a mapping. A point  $x \in X$  is an explosion point if every neighbor of  $x$  contains a subcontinuum  $C$  such that  $f^n(C) = X$  for some integer  $n$ . We will investigate the relationship between functions with indecomposable inverse limits and functions with explosion points.

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## Aposyndesis in plane continua without disjoint subcontinua with nonvoid interiors

Eldon Vought

This is joint work with Ned Grace. Let  $X$  be a plane continuum that has the property that every two subcontinua with nonvoid interiors intersect. It is proved that  $X$  is aposyndetic at no more than one point; more specifically, if  $X$  is aposyndetic at a point  $p$ , and  $x \in X \setminus \{p\}$ , then  $X$  is not aposyndetic at  $x$  with respect to  $p$ .