



**May 30, 31, June 1, 2002**

This conference, sponsored by California State University, Chico, will feature five one-hour talks. These speakers are Alejandro Illanes, Mark Marsh, Chris Mouron, Janusz Prajs, and David Ryden. There will be sessions for contributed papers on all three days.

To be included in the program, hard copy abstracts of contributed papers should be sent by April 20. Please send abstracts to Eldon J. Vought, Department of Mathematics and Statistics, California State University, Chico; Chico, California 95929.

A buffet supper will be held on Friday evening for all participants. And for those interested, some hiking on Saturday afternoon can be planned.

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**California State University, Chico**  
**Chico, California**

## ON THE COMPACTNESS THEOREM FOR SEQUENCES OF CLOSED SETS

Gerald Beer

We display the equivalence of the standard compactness theorem for sequences of closed sets in a separable metric space with a version of the Arzela-Ascoli Theorem valid in that setting. This in turn leads to a study of pointwise convergence of distance functions more generally and its relation to both Kuratowski convergence and Wijsman convergence.

## MAPPINGS CONFLUENT OVER LOCALLY CONNECTED CONTINUA

Janusz J. Charatonik

A mapping is said to be confluent over locally connected continua if for each locally connected subcontinuum  $Q$  of the range each component of its preimage is mapped onto  $Q$ . For mappings of compact spaces this class is a very natural generalization of locally confluent mappings. Various properties of these mappings, as relations to other known classes of mappings, composition properties, product properties, and limit properties are studied in the paper.

## INDUCED UNIVERSAL MAPPINGS

Raúl Escobedo

Each mapping  $f : X \rightarrow Y$  between continua induces a mapping  $\hat{f} : C(X) \rightarrow C(Y)$  between the corresponding hyperspaces of subcontinua, defined by  $\hat{f}(A) = f(A)$  for all  $A \in C(X)$ . The mapping  $f$  is said to be universal provided that for any mapping  $g : X \rightarrow Y$  there is a point  $p \in X$  such that  $f(p) = g(p)$ . A continuum  $X$  is said to be in  $Class(\hat{U})$  (Class  $(W)$ , respectively) provided that for any mapping from any continuum onto  $X$  the induced mapping is universal (onto, respectively). A continuum  $X$  is circle-like provided that for each  $\epsilon > 0$  there is a mapping  $f : X \rightarrow S^1$  such that  $diameter f^{-1}(f(x)) < \epsilon$  for all  $x \in X$ , where  $S^1$  denotes the unit circle in the plane. In this talk we prove the following result:

Theorem. Let  $X$  be a circle-like continuum. Then  $X$  is in  $Class(\hat{U})$  if and only if  $X$  is in  $Class(W)$ .

## HYPERSPACES OF DENDRITES

David Herrera-Carrasco

We prove the following theorem: Let  $X$  and  $Y$  be dendrites, assume that the set  $E(X)$  of end points of  $X$  is closed in  $X$ . If  $C(X) \approx C(Y)$ , then  $X \approx Y$ .

If  $E(X)$  is not closed, the assertion is not true.

## SPIRALS ON THE DOUBLE WARSAW CIRCLE

Charles Hagopian

We will discuss several open fixed-point problems involving spirals on the double Warsaw circle.

## THE HYPERSPACE $C_n(X)$ OF A CONTINUUM $X$

Alejandro Illanes

Let  $X$  denote a metric continuum. Let  $C_n(X)$  be the hyperspace defined by  $C_n(X) = \{A \subset X : A \text{ is closed, nonempty and } A \text{ has at most } n \text{ components}\}$ . There are several basic problems on  $C_n(X)$  which are not solved. For example:

-if  $S^1$  is the unit circle in the plane and  $n \geq 3$ , can  $C_n(S^1)$  be homeomorphic to  $C_n([0,1])$ ? *No. proved in his talk.*

-if  $n \neq m$  and  $C_n(X)$  is finite dimensional, can  $C_n(X)$  be homeomorphic to  $C_m(Y)$  for some continuum  $Y$ ?

-give a necessary and sufficient condition on a continuum  $X$  and an element  $A \in C_n(X)$  in order that  $C_n(X)$  is locally connected at  $A$ .

In this talk we discuss the known results and more questions related to these problems.

## **COMPACTNESS AND LOCAL COMPACTNESS IN HYPERSPACES**

**Camillo Costantini, Sandro Levi\*, Jan Pelant**

We give characterizations for a subspace of a hyperspace, endowed with either the Vietoris or Wijsman topology, to be compact or relatively compact. Then we characterize – both globally and at the single points – the local compactness of a hyperspace, endowed with either the Vietoris, the Wijsman, or the Hausdorff metric topology. Several examples illustrating the behavior of local compactness are also included.

## **SOME MAPPING THEOREMS FOR EXTENSIONAL DIMENSION**

**Wayne Lewis**

We present some results related to theorems of Pasynkov and Toruńczyk on the geometry of maps of finite dimensional compacta.

This represents work with Michael Levin while visiting at Texas Tech University.

## **PRODUCTS WITH THE FIXED POINT PROPERTY**

**Fernando Macías-Romero**

In this talk we prove the following theorem: Let  $X_1$  and  $X_2$  be continua with zero surjective semispan. If  $Y_1$  and  $Y_2$  are continua and  $f_1 : Y_1 \rightarrow X_1$  and  $f_2 : Y_2 \rightarrow X_2$  are any two surjective mappings, then the function  $f_1 \times f_2 : Y_1 \times Y_2 \rightarrow X_1 \times X_2$  defined by  $(f_1 \times f_2)(x_1, x_2) = (f_1(x_1), f_2(x_2))$  is a universal mapping. Hence  $X_1 \times X_2$  has the fixed point property.

## FANS WHOSE HYPERSPACE OF SUBCONTINUA ARE CONES

Sergio Macías\* and Sam B. Nadler, Jr.

A *dendroid* is an arcwise connected continuum such that the intersection of any two of its subcontinua is connected. A *fan* is a dendroid with only one ramification point.

Given a fan  $F$ , its *hyperspace of subcontinua* is:

$$C(F) = \{ A \subset X \mid A \text{ is a continuum} \},$$

with the Hausdorff metric.

We outline the proof of the following result:

**Theorem.** *If  $F$  is a fan, then the hyperspace of subcontinua of  $F$  is homeomorphic to a cone over a continuum if and only if  $F$  is homeomorphic to a cone over a compact metric space.*

## INVERSE LIMITS ON A POLYHEDRON AND THE FIXED POINT PROPERTY

Mark Marsh

Let  $P$  be a polyhedron and let  $X$  be an inverse limit on  $P$ . If  $P$  has the fixed point property (fpp), we consider the following questions.

1. For which  $P$  will  $X$  have the fpp?
2. If, for a particular  $P$ ,  $X$  doesn't have the fpp, what conditions can be added to the inverse limit space so that  $X$  will have the fpp?

We give a little history associated with these questions and we focus on a recent result of the speaker's, namely, if  $P$  is an even-dimensional real projective space and the bonding maps in the inverse limit system are homotopically essential, then  $X$  has the fpp.

## CONTINUA DETERMINED BY THEIR SUBCONTINUA

Verónica Martínez de la Vega y Mansilla

Let  $X$  be a metric continua. We say that  $X$  is *determined by its subcontinua* if it is possible to reconstruct the topology of  $X$  only by knowing all its subcontinua. The proper definition is:

A continuum  $X$  is determined by its subcontinua provided that  $X$  satisfies the following condition

(\*) if  $Y$  is a continuum and  $f : X \rightarrow Y$  is a bijection with the property that  $f(A)$  is a subcontinuum of  $Y$  if and only if  $A$  is a subcontinuum of  $X$ , then  $f$  is a *homeomorphism*.

In this talk we give examples, results and questions related to continua determined by their subcontinua.

## SEIFERT SURGERIES AND THE (-3,3,5) PRETZEL KNOT

Thomas Mattman,\* Katura Miyazaki, Kimihiko Motegi

The (-3,3,5) pretzel knot is the first known example of a non-invertible knot which admits a Seifert fibered Dehn filling. Therefore, the Seifert filling does not come from Dean's Primitive/Seifert construction. We discuss the significance of this discovery and how the (-3,3,5) knot can be used to generate an infinite family of non-invertible, Seifert fillable knots.

## AN INTRODUCTION TO A NEW CLASS OF HYPERSPACES

Eric L. McDowell\* and B. E. Wilder

Let  $X$  be a continuum with metric  $\rho$  and define  $C_\epsilon(X)$  to be the collection of those subcontinua of  $X$  that have  $\rho$ -diameter  $\leq \epsilon$ . We think of the resulting spaces as being approximations of  $X$  in much the same way that the mark of a pencil on paper approximates a point. In this talk, we will present results that we have discovered regarding the structure of the members of this new and interesting class of hyperspaces.

## **ON INVERSE LIMITS OF PIECEWISE LINEAR FUNCTIONS OF THE INTERVAL**

**Héctor Méndez-Lango**

A piecewise linear function of the interval  $I = [0, 1]$ ,  $f: I \rightarrow I$ , is in the family  $\mathcal{L}$  if the slope on each of the line segments of its graph is greater than one in absolute value. In this talk we show that if  $f \in \mathcal{L}$ , then the inverse limit space produced by taking  $f$  as the only bonding map contains an indecomposable subcontinuum.

## **ENTROPY AND CONTINUUM THEORY**

**Chris Mouron**

Topological entropy gives a measure of the chaotic behavior of a function. We will look at several equivalent definitions of entropy. Also, it is apparent that in order for a continuum to admit a positive entropy homeomorphism, some complex structure, such as indecomposability, must be present. Some theorems and examples will be given that verify this. Finally, the connection to entropy and expansive homeomorphisms will be made and applied.

## **TWO -TO-ONE MAPS ONTO HEREDITARILY DECOMPOSABLE CONTINUA**

**Van C. Nall**

It is not known whether there is a hereditarily decomposable tree-like continuum that is the image of a 2-1 map. However, if a hereditarily decomposable continuum is not tree-like, then it is the image of a 2-to-1 retraction. In fact, it is not known if an hereditarily decomposable uniquely arc connected continuum, that is a dendroid, is the image of a 2-to-1 map. We have shown that a dendroid is not the image of a 2-to-1 map from a hereditarily decomposable continuum.

## **A SPECIAL FAN**

**Felix Capulín Pérez**

In this talk we give an example of a fan which allows us to answer the two following questions:

Q.1 (T. J. Lee, 1991) Assume a fan  $X$  does not have the bend intersection property. Is  $X$  a type  $N$  continuum?

Q.2 (J. J. Charatonik, 1991) If a fan  $X$  admits no selection, is  $X$  a type  $N$  continuum?

## **INVERSE LIMITS OF COMPACTA CONVERGING IN COMPLETE METRIC SPACES**

**Janusz R. Prajs**

This talk is related to the Anderson-Choquet theorem about an inverse limit whose axes are embedded in a single metric space, and the inverse limit space that is homeomorphic to the topological limit of the embedded axes. In my recent research, variations of this theorem appeared in a natural way, several times independently. In my talk I intend to present a number of examples of applications of the technique involved in this theorem, and to show that it is an important tool in topology, particularly in continuum theory.

## **A SURVEY OF IRREDUCIBILITY AND INDECOMPOSABILITY IN METRIC CONTINUA**

**David J. Ryden**

This talk will begin with a survey of some classical results in irreducibility and indecomposability. In particular we will discuss a longstanding question recently answered in the negative by Solecki: does an indecomposable continuum  $I$  have a Borel transversal, which is to say a Borel subset that contains exactly one point from each component of  $I$ ? Solecki showed that the component equivalence relation of each indecomposable continuum is Borel bireducible with one of two canonical forms,  $\mathbb{E}_0$  and  $\mathbb{E}_1$ , both of which are too complicated to allow a Borel transversal.

We will examine these two categories as they manifest themselves in inverse limits of intervals by discussing some canonical examples and showing that a certain family of Markov maps always generates inverse limits of the simpler  $\mathbb{E}_0$  type. Whether or not all Markov maps do so is an open question.



## A CUTPOINT THEOREM FOR PLANE CONTINUA

E. E. Grace and Eldon Vought\*

Theorem. If  $M$  is a plane continuum with the property that every pair of subcontinua with nonvoid interiors intersect, then  $M$  has a weak cutpoint. This result answers in the affirmative a question posed by Howard Cook.

## MORE TOPOLOGICAL EXOTICA: REALLY LONG LINES

L. E. Ward

At the seventh Chico Topology Conference we defined a long line (LL) and showed that such an object exists and is unique.

Here we elaborate on that result by introducing the notion of a really long line (RLL). A RLL is a non-separable connected ordered space possessing neither a first nor a last element, and which is homogeneous in the order-theoretic sense. As a consequence, for each element  $x$  of a RLL, the rays  $(-\infty, x]$  and  $[x, +\infty)$  are homeomorphic.

Let  $I = [0, 1]$  and let  $I^\alpha$  be ordered lexicographically, where  $\alpha$  is an ordinal number. Then  $I^\alpha$  is an ordered continuum. Let  $X_\alpha$  denote  $I^\alpha$  with its endpoints deleted. We show that  $X_\alpha$  is a RLL if and only if  $\alpha$  is a countable ordinal that is not the sum of two smaller ordinals.



- 2:30 – 2:50 p.m. “Fans whose hyperspaces of subcontinua are cones”  
Sergio Macías (with Sam B.Nadler),  
 Universidad Nacional Autónoma de México
- 3:00 – 3:20 p.m. “Two-to-one maps onto hereditarily decomposable continua”  
Van C. Nall, University of Richmond
- 3:30 – 3:50 p.m. “A special fan”  
Felix Capulín Pérez, Universidad Nacional Autónoma de México
- 4:00 – 4:20 p.m. “Spirals on the double Warsaw circle”  
Charles Hagopian, California State University, Sacramento

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Friday, May 31 – Holt 185

- 9:30 – 10:20 a.m. “The hyperspace  $C_n(X)$  of a continuum  $X$ ”  
Alejandro Illanes, Universidad Nacional Autónoma de México
- 10:30 – 10:50 a.m. “Mappings confluent over locally connected continua”  
Janusz J. Charatonik, Universidad Nacional Autónoma de México
- 11:00 – 11:20 a.m. “A cutpoint theorem for plane continua”  
Eldon Vought (with E.E. Grace),  
 California State University, Chico.
- 11:30 – 11:50 a.m. “Products with the fixed point property”  
Fernando Macías – Romero,  
 Universidad Nacional Autónoma de Puebla

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- 1:30 – 2:20 p.m. “A survey of irreducibility and indecomposability in metric continua”  
David J. Ryden, Tulane University
- 2:30 – 2:50 p.m. “On inverse limits of piecewise linear functions of the interval”  
Héctor Méndez-Lango, Universidad Nacional Autónoma de México
- 3:00 – 3:20 p.m. “An introduction to a new class of hyperspaces”  
Eric McDowell (with B.E. Wilder), Berry College
- 3:30 – 3:50 p.m. “Continua determined by their subcontinua”  
Verónica Martínez de la Vega y Mansilla, Universidad Nacional Autónoma de México
- 4:00 – 4:20 p.m. “Induced universal mappings”  
Raúl Escobedo, Universidad Nacional Autónoma de Puebla

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Saturday, June 1 – Holt 185

- 9:20 – 10:10 a.m. “Inverse limits on a polyhedron and the fixed point property”  
Mark Marsh, California State University, Sacramento
- 10:20 – 10:40 a.m. “Seifert surgeries and the  $(-3,3,5)$  pretzel knot”  
Thomas Mattman (with Katura Miyazaki, Kimihiko Motegi), California State University, Chico

10:50 - 11:10 a.m. "Compactness and local compactness in  
hyperspaces"

Sandro Levi (with Camillo  
Costantini, Jan Pelant), Università  
di Milano-Bicocca

11:20 - 11:40 a.m. "To be announced"

Harold Bell, University of Cincinnati

11:50 - 12:10 p.m. "To be announced"

Jerry Beer, California State  
University, Los Angeles

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